PERTH COLLEGE
YR 12 3CD SPECIALIST MATHEMATICS
SEMESTER ONE 2010
TEST 2

Name: **Time : 50 mins** **Total marks : /45 = %**

- Answer all questions neatly in the spaces provided.
- **Show all working** where appropriate.
- **Calculator & Formula Sheet allowed**

1) (3, 3 marks)

If $x = 3t^2 + 4t$ and $y = \frac{1}{t+1}$ find

a) $\frac{dy}{dx}$ in terms of t ,

$$\frac{dy}{dt} = \frac{-1}{(t+1)^2} \qquad \frac{dx}{dt} = 6t + 4$$

$$\frac{dy}{dx} = \frac{-1}{(t+1)^2} \cdot \frac{1}{6t+4} = \frac{-1}{(t+1)^2(6t+4)}$$

b) $\frac{d^2y}{dx^2}$ in terms of t

$$\frac{dy}{dx} = (-1)(t+1)^{-2}(6t+4)^{-1}$$

$$\frac{d^2y}{dx^2} = (-1)(-2)(t+1)^{-3} \cdot \frac{dt}{dx}(6t+4)^{-1} + (-1)(t+1)^{-2}(-1)(6t+4)^{-2}(6)\frac{d}{d}$$

$$\frac{d^2y}{dx^2} = 2(t+1)^{-3}(6t+4)^{-1}(6t+4)^{-1} + 6(t+1)^{-2} \cdot (6t+4)^{-2} \cdot (6t+4)^{-}$$

$$= \frac{2}{(t+1)^3 \cdot (6t+4)^2} + \frac{6}{(t+1)^2(6t+4)^3} = \frac{18t+14}{(t+1)^3(6t+4)^3}$$

2) (2, 3, 3, 4, 6 marks)

Determine the following integrals. Show appropriate working for **full marks.**

a) $\int 3x^2 + \sin \pi x \; dx$

$$x^3 - \frac{\cos \pi x}{\pi} + c$$

b) $\int \frac{5x^2 - 3x}{\sqrt{x}} \; dx$

$$= \int 5x^{3/2} - 3x^{1/2} \, dx$$

$$= \frac{5x^{5/2}}{5/2} - \frac{3x^{3/2}}{3/2} + c$$

$$= 2x^{5/2} - 2x^{3/2} + c$$

c) $\int 6x^2 \sqrt{(1 + x^3)} \; dx$

Working

Guess $y = (1 + x^3)^{3/2}$

$$y' = \frac{3}{2}(1 + x^3)^{1/2} 3x^2$$

$$= \frac{9x^2 (1 + x^3)^{1/2}}{2}$$

adjust

$$y = \frac{4}{3}(1 + x^3)^{3/2} + c$$

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d) $\int \cos^2 x \sin^3 x \, dx$

$$= \int \cos^2 x \sin^2 x \sin x$$

$$= \int \cos^2 x \left(1-\cos^2 x\right) \sin x \, dx$$

$$= \int \cos^2 x \sin x - \cos^4 x \sin x \, dx$$

$$= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$$

e) $\int \sin^4 x \, dx$

$$= \int \sin^2 x \sin^2 x \, dx$$

$$\cos 2x = 1 - 2\sin^2 x$$
$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right) dx$$

$$= \int \frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x \, dx$$

$$= \int \frac{1}{4} - \frac{1}{2}\cos 2x + \left(\frac{1}{4}\right)\frac{1+\cos 4x}{2} \, dx$$

$$= \frac{x}{4} - \frac{\sin 2x}{4} + \frac{x}{8} + \frac{\sin 4x}{32} + C$$

$$= \frac{3}{8}x - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + c$$

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3) (5, 5 marks)

Perform the following integrations **using the given substitutions**. Show all working.

a) $\int \frac{4x}{\sqrt{4-x^2}}\,dx$ let $x = 2\sin\theta$

$$\frac{dx}{d\theta} = 2\cos\theta$$

$$dx = 2\cos\theta\, d\theta$$

$$\int \frac{4x}{\sqrt{4-x^2}}\,dx$$

$$= \int \frac{4.2(\sin\theta)}{\sqrt{4-(2\sin\theta)^2}}\,dx$$

$$= \int \frac{8\sin\theta}{\sqrt{4-4\sin^2\theta}} \cdot 2\cos\theta\,d\theta$$

$$= \int \frac{8\sin\theta}{\sqrt{4(1-\sin^2\theta)}} \cdot 2\cos\theta\,d\theta$$

$$= \int \frac{8\sin\theta}{2\cos\theta} \cdot 2\cos\theta\,d\theta$$

$$= \cos\theta + C$$

b) $\int \frac{4x}{\sqrt{x-3}}\,dx$ let $u = x-3$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$x = u+3$$

$$\int \frac{4(u+3)}{\sqrt{u}} \cdot du$$

$$= \int \frac{4u+12}{u^{1/2}} \cdot du$$

$$= \int 4u^{1/2} + 12u^{-1/2} \cdot du$$

$$= \frac{4u^{3/2}}{3/2} + \frac{12u^{1/2}}{1/2} + c = \frac{8}{3}(x-3)^{3/2} + 24(x-3)^{1/2} + c$$

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4) (4 marks)

The equation of the gradient to a curve is given by

$g'(x) = 4 + \frac{3}{x^2} + 2\pi \cos \pi x$

If the point (1,2) lies on the curve, find the equation of g(x)

$$g(x) = 4x - 3x^{-1} + 2\pi \frac{\sin \pi x}{\pi} + c$$

$$g(x) = 4x - \frac{3}{x} + 2\sin \pi x + c$$

$$(1,2)$$

$$2 = 4(1) - \frac{3}{1} + 2\sin \pi + c$$

$$\therefore \quad c = 1$$

$$g(x) = 4x - \frac{3}{x} + 2\sin \pi x + 1$$

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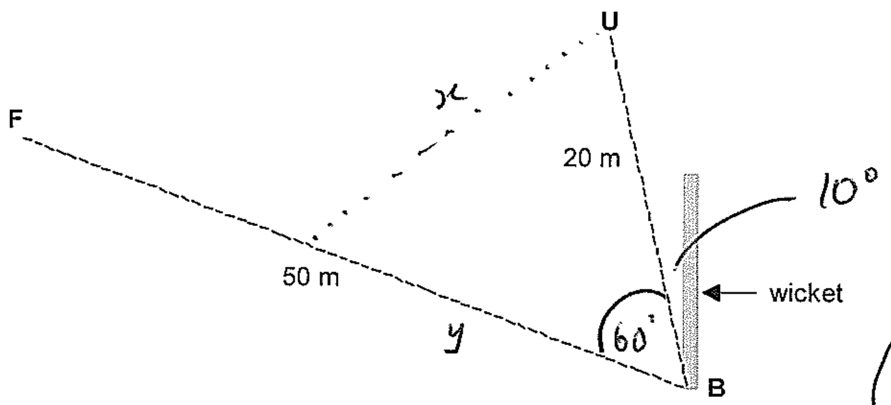
5 ~~6~~ **(7 marks)**

In a cricket match, a ball is hit from B at a constant speed of 30 ms^{-1} towards a fieldsperson at F, located 50 metres from B along the bearing 290°.

An umpire is standing at U, 20 metres from B along bearing 350°.

Determine how fast the distance between the ball and the umpire is changing when the ball has travelled half-way towards the fieldsperson. Assume that the fieldsperson and the umpire have not moved from their respective positions.

diag.



NOT DRAWN TO SCALE

$\frac{dy}{dt} = 30$ m/s

Find $\frac{dx}{dt} =$

x = distance from umpire to ball

y = distance ball is hit

$x^2 = y^2 + 20^2 - 2.y.20\cos(60°)$

$x^2 = y^2 - 20y + 400$

$2x.\frac{dx}{dt} = 2y\frac{dy}{dt} - 20\frac{dy}{dt}$

$\frac{dx}{dt} = \frac{dy}{dt}\left(\frac{y-10}{x}\right)$

at $y=25$

$x^2 = \sqrt{25^2 - 20.25 + 400}$

$x = \sqrt{525}$ m

*

at $y=25$

$\frac{dx}{dt} = 30\frac{(25-10)}{\sqrt{525}} = 19.64$ m/sec.

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